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A hydrodynamic formulation of the evolution of large-scale structure in the Universe is presented. It relies on the spatially coarse-grained description of the dynamical evolution of a many-body gravitating system. Because of the assumed irrelevance of short-range (“collisional”) interactions, the way to tackle the hydrodynamic equations is essentially different from the usual case. The main assumption is that the influence of the small scales over the large-scale evolution is weak: this idea is implemented in the form of a *large-scale expansion* for the coarse-grained equations. This expansion builds a framework in which to *derive* in a controlled manner the popular “dust” model (as the lowest-order term) and the “adhesion” model (as the first-order correction). It provides a clear physical interpretation of the assumptions involved in these models and also the possibility to improve over them.

98.80.-k, 98.80.Hw, 98.65.Dx, 04.40.-b

I. INTRODUCTION

The standard model to understand the large-scale features of the matter distribution in the Universe after decoupling from radiation can be hardly simpler: a collection of many identical point particles interacting with each other via the Newtonian gravitational force in an expanding spatial background [1,2]. Structure arises as a consequence of the gravitational instability of initially tiny density perturbations. This model neglects relativistic effects, which become important only at scales of the order of the horizon and beyond, or when dealing with relativistic velocities. The model also excludes nongravitational interactions, which are assumed to be relevant only at small enough scales.

The general solution to the dynamical evolution of this model is unknown due to the mathematical difficulties. N-body simulations, which numerically solve the dynamical equations (see Eqs. (1) in the next section), have been very helpful in understanding this evolution. In this work I look for an analytical derivation of some of the relevant features in the formation of large-scale structures. One is not usually interested in following the detailed path of each particle, but rather in some general properties that typically depend on the behavior of a large number of particles and which in the end are the kind of data provided by observations, e.g., the smoothed density and velocity fields. This naturally leads to the use of a *coarse-grained* or smoothed description: the evolution of a few, “macroscopic” variables is isolated by making suitable approximations for the evolution of the neglected degrees of freedom and for their influence on the relevant ones.

The idea of a spatial coarse-graining has a long history in the field of statistical mechanics, where it has proven quite successful as a powerful tool for extracting information about the dynamical evolution of many-body systems. In this work I explore its application in the context of cosmological structure formation. Although the procedure of smoothing a field is widely used in cosmology, this application has mainly had a descriptive purpose. Unlike this, I study systematically the *dynamical* evolution of the smoothed fields. As we shall see, this method has the merit of providing a common framework for different models (dust and adhesion) of structure formation: it offers a clear explanation of the approximations involved in each model and of their physical meaning, so that it also opens the way to systematically relax them and obtain improved models. Starting from the microscopic equations of motion of the particles, I derive an exact set of hydrodynamic-like equations for the (coarse-grained) mass-density and velocity fields (Sec. II). These equations do not form an autonomous set, but rather constitute the first ones of an infinite hierarchy that must be truncated to become a useful tool. I discuss in Sec. III how this can be achieved by resort to a *large-scale expansion*, based on the assumption that the dynamical influence of the small scales over the large-scale evolution is weak. The lowest-order term of the expansion yields the dust model (Sec. IV), and it corresponds to a complete neglect of the structure below the coarsening scale. This leads to an eventual failure of the model (in the

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form of pancake-like singularities), which can be prevented by considering the first-order correction in the expansion (Sec. V): it accounts for the dynamical influence of the structure below the coarsening scale, and I show by means of boundary-layer techniques that it gives rise to a robust “adhesive” behavior of the same kind as the adhesion model. I end up in Sec. VI with a discussion of the results.

II. COARSE-GRAINING THE BASIC EQUATIONS

The basic model is a system of nonrelativistic, identical point particles which (i) are assumed to interact with each other via gravity only; (ii) look homogeneously distributed on sufficiently large scales, so that the evolution corresponds to an expanding Friedmann-Lemaître cosmological background; and (iii) deviations to homogeneity are relevant only on scales small enough that a Newtonian approximation is valid to follow their evolution. Let $a(t)$ denote the expansion factor of the Friedmann-Lemaître cosmological background, $H(t) = \dot{a}/a$ the associated Hubble function, and $\varrho_b(t)$ the homogeneous (background) density on large scales. \mathbf{x}_i is the comoving spatial coordinate of the i -th particle, \mathbf{u}_i is its peculiar velocity, and m its mass. In terms of these variables the evolution is described by the following set of equations [1] (∇_i denotes a partial derivative with respect to \mathbf{x}_i):

$$\dot{\mathbf{x}}_i = \frac{1}{a} \mathbf{u}_i, \quad (1a)$$

$$\dot{\mathbf{u}}_i = \mathbf{w}_i - H \mathbf{u}_i, \quad (1b)$$

$$\nabla_i \cdot \mathbf{w}_i = -4\pi G a \left[\frac{m}{a^3} \sum_{j \neq i} \delta(\mathbf{x}_i - \mathbf{x}_j) - \varrho_b \right], \quad (1c)$$

$$\nabla_i \times \mathbf{w}_i = \mathbf{0}, \quad (1d)$$

where \mathbf{w}_i is the peculiar gravitational acceleration acting on the i -th particle. Finally, Eqs. (1) must be subjected to periodic boundary conditions in order to yield a Newtonian description consistent with the Friedmann-Lemaître solution at large scales [3].

To implement the idea of a spatial coarse-graining one employs a smoothing window $W(z)$: this smoothing window should define a bounded region of space and whose inner structure is smoothed out. In App. A I discuss the general properties I will require from a smoothing window. Let L denote the (comoving) coarse-graining length scale. A microscopic mass density field and a coarse-grained mass density field can be defined respectively as follows:

$$\varrho_{mic}(\mathbf{x}, t) = \frac{m}{a(t)^3} \sum_i \delta^{(3)}(\mathbf{x} - \mathbf{x}_i(t)), \quad (2a)$$

$$\varrho(\mathbf{x}, t; L) = \int \frac{d\mathbf{y}}{L^3} W\left(\frac{|\mathbf{x} - \mathbf{y}|}{L}\right) \varrho_{mic}(\mathbf{y}, t). \quad (2b)$$

The physical interpretation of the field $\varrho(\mathbf{x}; L)$ follows straightforwardly from the properties of the smoothing window: it is proportional to the number of particles contained within the coarsening cell of size $\approx L$ centered at \mathbf{x} . A microscopic peculiar-momentum density field and the corresponding coarse-grained field can be defined in the same way:

$$\mathbf{j}_{mic}(\mathbf{x}, t) = \frac{m}{a(t)^3} \sum_i \mathbf{u}_i(t) \delta^{(3)}(\mathbf{x} - \mathbf{x}_i(t)), \quad (3a)$$

$$\mathbf{j}(\mathbf{x}, t; L) = \int \frac{d\mathbf{y}}{L^3} W\left(\frac{|\mathbf{x} - \mathbf{y}|}{L}\right) \mathbf{j}_{mic}(\mathbf{y}, t). \quad (3b)$$

One can introduce peculiar velocity fields \mathbf{u}_{mic} and \mathbf{u} by definition as $\mathbf{j} = \varrho \mathbf{u}$ and similarly for \mathbf{u}_{mic} . The physical meaning of $\mathbf{u}(\mathbf{x}; L)$ is also simple: it is the center-of-mass peculiar velocity of the subsystem defined by the particles

inside the coarsening cell of size $\approx L$ centered at \mathbf{x} . Notice that \mathbf{u} is *not* obtained by coarse-graining \mathbf{u}_{mic} : from a dynamical point of view, it is more natural to coarse-grain the momentum rather than the velocity, since the former is an additive quantity for a system of particles. Finally, one can define peculiar gravitational acceleration fields \mathbf{w}_{mic} and \mathbf{w} through a coarse-graining of the force:

$$\varrho_{mic} \mathbf{w}_{mic}(\mathbf{x}, t) = \frac{m}{a(t)^3} \sum_i \mathbf{w}_i(t) \delta^{(3)}(\mathbf{x} - \mathbf{x}_i(t)), \quad (4a)$$

$$\varrho \mathbf{w}(\mathbf{x}, t; L) = \int \frac{d\mathbf{y}}{L^3} W\left(\frac{|\mathbf{x} - \mathbf{y}|}{L}\right) \varrho_{mic} \mathbf{w}_{mic}(\mathbf{y}, t). \quad (4b)$$

The field $\mathbf{w}(\mathbf{x})$ has the physical meaning of the center-of-mass peculiar gravitational acceleration of the subsystem defined by the coarsening cell at \mathbf{x} .

From these definitions and Eqs. (1a) and (1b), it is straightforward to derive the evolution equations obeyed by the coarse-grained fields ϱ and \mathbf{u} (from now on, $\partial/\partial t$ is taken at constant \mathbf{x} , and ∇ means partial derivative with respect to \mathbf{x}):

$$\frac{\partial \varrho}{\partial t} + 3H\varrho = -\frac{1}{a} \nabla \cdot (\varrho \mathbf{u}), \quad (5a)$$

$$\frac{\partial(\varrho \mathbf{u})}{\partial t} + 4H\varrho \mathbf{u} = \varrho \mathbf{w} - \frac{1}{a} \nabla \cdot (\varrho \mathbf{u} \mathbf{u} + \Pi), \quad (5b)$$

where a new second-rank tensor field has been defined:

$$\Pi(\mathbf{x}, t; L) = \int \frac{d\mathbf{y}}{L^3} W\left(\frac{|\mathbf{x} - \mathbf{y}|}{L}\right) \varrho_{mic}(\mathbf{y}, t) [\mathbf{u}_{mic}(\mathbf{y}, t) - \mathbf{u}(\mathbf{x}, t; L)][\mathbf{u}_{mic}(\mathbf{y}, t) - \mathbf{u}(\mathbf{x}, t; L)]. \quad (6)$$

The field $\Pi(\mathbf{x})$ is due to the velocity dispersion, i.e., to the fact that the particles in the coarsening cell have in general a velocity different from that of the center of mass. The trace of Π is proportional to the internal kinetic energy of the coarsening cell, that is, the total kinetic energy of the particles in the reference frame of the center of mass.

The physical meaning of Eqs. (5a) and (5b) is simple: they are just balance equations, stating mass conservation and momentum conservation, respectively. The term $\nabla \cdot \Pi$ represents a *kinetic* pressure due to the exchange of particles between neighboring coarsening cells (just like in the ideal gas) and it has the same physical origin as the convective term $\nabla \cdot (\varrho \mathbf{u} \mathbf{u})$, i.e., a nonlinear mode-mode coupling of the velocity field. The difference is that the convective term couples only modes on scales $> L$, while the velocity dispersion term codifies the dynamical effect of the coupling of the modes on scales $> L$ with the modes on scales $< L$. The term $\varrho \mathbf{w}$ codifies the gravitational interaction between the coarsening cells and it is shown later that it can be split in a similar manner into a contribution due to the large scales and another due to the coupling of the large scales with the small ones. Although Eqs. (5) look similar to the ordinary hydrodynamic equations, there is the important difference that these equations are *exact*: as one changes the smoothing length, the fields ϱ , \mathbf{u} , \mathbf{w} , Π change but in such a way that the equations remain the same (for example, upon increase of the smoothing length, part of the dynamical effect described by $\nabla \cdot (\varrho \mathbf{u} \mathbf{u})$ is shifted towards $\nabla \cdot \Pi$). This property is reflected in that the equations are not an autonomous system for ϱ and \mathbf{u} . In fact, they are just the first ones of an infinite hierarchy, as can be checked by computing the evolution equations for the fields \mathbf{w} and Π (see Eq. (B1), for example). To obtain a useful set of equations, it is necessary to truncate this hierarchy by looking for a functional dependence of \mathbf{w} and Π on ϱ and \mathbf{u} . This will be the task of the next section.

III. THE LARGE-SCALE EXPANSION

In this section I discuss the closure of the hydrodynamic hierarchy at the level of Eqs. (5). When the particle interaction is dominated by a fast-decaying (“collisional”) force, as in normal fluids, this truncation is achieved by the assumption of local equilibrium (see, e.g., Refs. [4,5]). In this case, the interaction determines a privileged smoothing scale L and it drives the coarsening cells of size L towards an approximate internal thermal equilibrium, as if isolated from each other. The evolution of the large scales ($\gg L$) is then ruled by the interaction between neighboring coarsening cells. In this way, for example, Eq. (5b) becomes Navier-Stokes’ equation.

One cannot, however, apply this approach to the problem in hand: a system with a long-ranged, unshielded interaction such as gravity does not obey the usual thermodynamics (see, e.g., the brief review in Ref. [6] and the

more technical remarks in Ref. [5]). Moreover, this long range implies that the interaction is self-similar and does not pick up itself a favored coarsening length. One must therefore make use of a different approach to close the hydrodynamic hierarchy.

For this purpose, I introduce the *large-scale expansion*, which relies on the assumption that, in the context of cosmological structure formation, the evolution of the large scales is weakly influenced by what is going on in the small scales. This assumption is further discussed in Sec. VI. Here I show how to formulate this idea in order to write the fields \mathbf{w} and Π in Eqs. (5) in terms of ϱ and \mathbf{u} . Let a tilde denote the Fourier transform of any field:

$$\tilde{\phi}(\mathbf{k}) = \int_V d\mathbf{x} e^{i\mathbf{k}\cdot\mathbf{x}} \phi(\mathbf{x}), \quad (7a)$$

$$\phi(\mathbf{x}) = \frac{1}{V} \sum_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}} \tilde{\phi}(\mathbf{k}), \quad (7b)$$

where V denotes the volume of periodicity for Eqs. (1). Then the definition (2b) can be written as $\tilde{\varrho}(\mathbf{k}; L) = \tilde{W}(L\mathbf{k}) \tilde{\varrho}_{mic}(\mathbf{k})$. Making use of the Taylor-expansion (A3) and formally inverting it, one gets:

$$\tilde{\varrho}_{mic}(\mathbf{k}) = [1 + \frac{1}{2}B(Lk)^2 + o(Lk)^4] \tilde{\varrho}(\mathbf{k}; L), \quad (8a)$$

$$\varrho_{mic}(\mathbf{x}) = [1 - \frac{1}{2}B(L\nabla)^2 + o(L\nabla)^4] \varrho(\mathbf{x}; L). \quad (8b)$$

Consider first the field \mathbf{w} . Fourier-transforming its definition (4) and using Eqs. (1c) and (1d), one finds:

$$\widetilde{\varrho\mathbf{w}}(\mathbf{k}; L) = -\frac{4\pi Ga}{V} \sum_{\mathbf{q} \neq \mathbf{0}} \frac{i\mathbf{q}}{q^2} \tilde{\varrho}_{mic}(\mathbf{q}) \tilde{\varrho}_{mic}(\mathbf{k} - \mathbf{q}) \tilde{W}(Lk). \quad (9)$$

This expression collects the mode-mode coupling via gravity of the scales $\sim k^{-1}$ with every other scale. Take now a large scale $k^{-1} \gg L$: the assumption of weak coupling to the small scales then means that in the summation in Eq. (9), the main contribution arises also from the large scales, $q^{-1}, |\mathbf{k} - \mathbf{q}|^{-1} \gg L$. One is then justified to insert the expansions (8a) and (A3) into Eq. (9). Transforming back into real space yields the result:

$$\varrho\mathbf{w} = \varrho\mathbf{w}^{mf} + BL^2(\nabla\varrho \cdot \nabla)\mathbf{w}^{mf} + o(L\nabla)^4, \quad (10a)$$

$$\nabla \cdot \mathbf{w}^{mf} = -4\pi Ga(\varrho - \varrho_b), \quad (10b)$$

$$\nabla \times \mathbf{w}^{mf} = \mathbf{0}. \quad (10c)$$

Thus, the unknown field \mathbf{w} has been written as a functional of the coarse-grained density field. The combination $L\nabla (\leftrightarrow Lk)$ can be *formally* viewed as a parameter which measures the influence of the small scales over the large scales and which has been assumed to be small: hence the name large-scale expansion. The field \mathbf{w}^{mf} , which can be called the *macroscopic* gravitational field, represents the gravitational field created by the monopole moment of the matter distribution in the coarsening cells, i.e., as if they had no spatial extension ($L = 0$). It obeys the equations that one would have naively guessed and we see that this is not the whole story: there exist a correction due to the higher multipole moments that can then be properly called *tidal* correction and it represents the coupling of the large to the small scales induced by gravity. Hence, this decomposition into a macroscopic field and a tidal correction is analogous to the decomposition in Eq. (5b) into convection, $\varrho\mathbf{u}\mathbf{u}$, and velocity dispersion, Π .

The same procedure can be now applied to the field Π . Now, $\mathbf{j}_{mic} = \varrho_{mic}\mathbf{u}_{mic}$ obeys an expansion like (8b), but \mathbf{u}_{mic} does not, because it is not defined by a straightforward coarse-graining. What one has in this case is slightly more involved:

$$\mathbf{u}_{mic} = \frac{\mathbf{j}_{mic}}{\varrho_{mic}} = \frac{[1 - \frac{1}{2}B(L\nabla)^2 + o(L\nabla)^4]\mathbf{j}}{[1 - \frac{1}{2}B(L\nabla)^2 + o(L\nabla)^4]\varrho} = \left[1 - \frac{1}{2}B(L\nabla)^2 - BL^2\frac{\nabla\varrho}{\varrho} \cdot \nabla + o(L\nabla)^4\right] \mathbf{u}. \quad (11)$$

Because of this minor complication, it is simpler in this case to work directly with the representation in real space, Eq. (6), into which the expansion (11) and the one corresponding to \mathbf{j}_{mic} are now inserted. Because the smoothing window

effectively restricts $|\mathbf{x} - \mathbf{y}| \lesssim L$, the fields within the integral evaluated at point \mathbf{y} can be consistently Taylor-expanded around point \mathbf{x} to order $(L \nabla)^4$. The final result reads:

$$\Pi = BL^2 \varrho(\partial_i \mathbf{u})(\partial_i \mathbf{u}) + o(L \nabla)^4, \quad (12)$$

where a summation over the index i is implied. Taking this result and (10) into the hydrodynamic hierarchy (5), and dropping the assumed small corrections $o(L \nabla)^4$, I finally obtain

$$\frac{\partial \varrho}{\partial t} + 3H\varrho = -\frac{1}{a} \nabla \cdot (\varrho \mathbf{u}), \quad (13a)$$

$$\frac{\partial(\varrho \mathbf{u})}{\partial t} + 4H\varrho \mathbf{u} = \varrho \mathbf{w}^{mf} - \frac{1}{a} \nabla \cdot (\varrho \mathbf{u} \mathbf{u}) + BL^2 \left\{ (\nabla \varrho \cdot \nabla) \mathbf{w}^{mf} - \frac{1}{a} \nabla \cdot [\varrho(\partial_i \mathbf{u})(\partial_i \mathbf{u})] \right\}, \quad (13b)$$

$$\nabla \cdot \mathbf{w}^{mf} = -4\pi G a (\varrho - \varrho_b), \quad (13c)$$

$$\nabla \times \mathbf{w}^{mf} = \mathbf{0}. \quad (13d)$$

This is an autonomous system of equations for the two coarse-grained fields ϱ and \mathbf{u} . Compared to the hydrodynamic equations of a normal fluid, we see that, because of the long range of the interaction, its contribution $\varrho \mathbf{w}$ to the equation for momentum conservation cannot be written as the divergence of a tensor, i.e., as a nonideal correction to the kinetic contribution represented by Π . Also the expression to lowest-order for Π is another evident difference. In the next sections, I explore the dynamical evolution described by Eqs. (13).

IV. THE DUST MODEL

In this section I consider the lowest order in the large-scale expansion. This corresponds to formally setting $L = 0$ in Eq. (13b), thus yielding:

$$\frac{\partial \varrho}{\partial t} + 3H\varrho + \frac{1}{a} \nabla \cdot (\varrho \mathbf{u}) = 0, \quad (14a)$$

$$\frac{\partial \mathbf{u}}{\partial t} + H\mathbf{u} + \frac{1}{a} (\mathbf{u} \cdot \nabla) \mathbf{u} = \mathbf{w}^{mf}, \quad (14b)$$

$$\nabla \cdot \mathbf{w}^{mf} = -4\pi G a (\varrho - \varrho_b), \quad (14c)$$

$$\nabla \times \mathbf{w}^{mf} = \mathbf{0}. \quad (14d)$$

This is the popular and thoroughly studied *dust model* (see, e.g., Refs. [1,2,7]). The large-scale expansion provides a clear picture of the approximations leading from Eqs. (1) to this model: by setting $L = 0$ one assumes that the coarsening cells can be thought of as “big particles” lacking completely an internal structure (Fig. 1). This implies neglecting (i) the velocity dispersion Π compared to the convection term $\varrho \mathbf{u} \mathbf{u}$; and (ii) the spatial extension of the cells and thus the tidal correction compared to the macroscopic gravitational field \mathbf{w}^{mf} .

It will be useful to briefly review some results for the dust model. In the linear regime of small fluctuations about the homogeneous cosmological background, Eqs. (14) can be linearized and the resulting set of linear differential equations solved. In particular, in the long-time limit (but still within the linear approximation), the peculiar velocity and the gravitational acceleration are related by the condition of parallelism:

$$\mathbf{w}^{mf}(\mathbf{x}, t) = F(t) \mathbf{u}(\mathbf{x}, t), \quad (15)$$

with the function

$$F(t) = 4\pi G \varrho_b(t) \frac{b(t)}{\dot{b}(t)} > 0, \quad (16)$$

where $b(t)$ is the growing mode of the (small) density contrast, i.e., the growing solution of the equation $\ddot{b} + 2H\dot{b} - 4\pi G\rho_b b = 0$.

The linear evolution eventually breaks down due to the growth of inhomogeneities by gravitational instability. The solution to the fully nonlinear equations (14) is not known, but a successful approximation in this regime is the *Zel'dovich approximation* (ZA hereafter) [7–9], which turns out to be an exact solution of Eqs. (14) for some highly symmetric configurations. In the most general case, it can be understood as the extrapolation of the parallelism condition (15) into Eq. (14b) [10]. More properly, considering the way Eqs. (14) were derived, the actual approximation is the *truncated ZA* [11], since the density and velocity fields have been smoothed on a scale L . Comparison with N-body simulations [11] shows that the truncated ZA performs substantially better than the original ZA: this is no wonder within the present coarse-graining formalism, since the particles that must obey the “dust” evolution (and thus be moved according to the ZA) are not the N-body particles of the simulations, which follow Eqs. (1), but rather the “big particles” which represent the coarsening cells.

If one introduces a rescaled velocity field $\mathbf{v} = \mathbf{u}/a\dot{b}$, and uses $b(t)$ as the new temporal variable, then Eq. (14b) becomes in the ZA:

$$\frac{\partial \mathbf{v}}{\partial b} + (\mathbf{v} \cdot \nabla) \mathbf{v} = 0, \quad (17a)$$

$$\nabla \times \mathbf{v} = \mathbf{0}, \quad (17b)$$

together with the irrotationality constraint following from Eqs. (15) and (14d). Hence, the problem reduces to the curl-free evolution of a fluid under no forcing at all. The solution to this equation is then inserted into the continuity equation (14a) to yield the evolved density field. As is well-known, however, the ZA has the problem of giving rise to singularities in the fields, mainly sheet-like singularities or “pancakes” [9] at which $\nabla \cdot \mathbf{v} \rightarrow -\infty$ and $\varrho \rightarrow +\infty$. Beyond this moment, the ZA ceases to be valid. The reason lies in the nonlinear, convective term in Eq. (17a), which deforms the initial velocity field and generically leads to a multivalued velocity field (*shell crossing* in the cosmological literature). This feature is likely not exclusive to the ZA but a generic property of the fully nonlinear dust model, Eqs. (14), as checked, e.g., by the application of Lagrangian perturbation techniques (see [7] and refs. therein). The generation of singularities by the convective term is not prevented by the gravitational acceleration \mathbf{w}^{mf} , which, on the contrary, favors this process. The emergence of singularities signals the unsuitability of the dust model beyond that time: in the next section I investigate how the correction to dust following from the large-scale expansion regularizes the singularities.

V. THE ADHESION MODEL

The singularities of the dust model arise because the approximation of the coarsening cells as “big particles” is bad when the density of these “particles” is large (formally infinite): the interaction between them is no longer the simple macroscopic gravitational force $\varrho \mathbf{w}^{mf}$, but it becomes more and more dependent on the internal structure of the coarsening cells. Therefore, a first step to improve the dust model is to take into account the first-order correction following from the large-scale expansion and study Eqs. (13).

Obviously, the nonlinearities represented by this correction make it even more difficult to solve these equations than in the case of the dust model. Hence, in order to understand the effects of the correction on the dust evolution, I introduce some further simplifying assumptions. The idea is to consider the limit $BL^2 \rightarrow 0^+$ in Eqs. (13), so that the correction is irrelevant everywhere *except at those places where the dust evolution would predict a singularity*. Therefore, the evolution will follow the ZA almost everywhere and the parallelism approximation (15) will be good: this implies in particular, via Eqs. (13c) and (13d), that \mathbf{u} is also curl-free and that $\nabla \cdot \mathbf{u} \propto -(\varrho - \varrho_b)$. Since the correction to dust in Eq. (13b) will be effective only near the singularities and these correspond in the ZA predominantly to pancakes, the correction can be computed under the approximation of a local plane-parallel collapse: ϱ and \mathbf{u} change only along the direction \mathbf{n} of the local plane-parallel collapse and $\partial_i u_j \approx (\nabla \cdot \mathbf{u}) n_i n_j$. Making use of these approximations, the correction term can be simplified as follows:

$$\frac{1}{\varrho} (\nabla \varrho \cdot \nabla) \mathbf{w}^{mf} \approx \frac{F}{\varrho} (\nabla \cdot \mathbf{u}) \nabla \varrho \approx F \nabla^2 \mathbf{u} \quad (\propto -\nabla \varrho), \quad (18a)$$

$$\frac{1}{a\varrho} \nabla \cdot [\varrho (\partial_i \mathbf{u}) (\partial_i \mathbf{u})] \approx \frac{1}{a\varrho} \nabla [\varrho (\nabla \cdot \mathbf{u})^2] \approx \frac{3}{a} (\nabla \cdot \mathbf{u}) \nabla^2 \mathbf{u} \quad (\propto -\varrho \nabla \varrho), \quad (18b)$$

where it has been taken into account that the correction will be effective only at those places where $\varrho \sim (-\nabla \cdot \mathbf{u}) \rightarrow +\infty$. In this limit, the contribution (18b) due to the velocity dispersion will be dominant over that of the tidal correction (18a). Inserting this result in Eq. (13b) and in terms of the variables \mathbf{v} and b previously introduced, I finally arrive at the following model:

$$\frac{\partial \mathbf{v}}{\partial b} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \nu |\nabla \cdot \mathbf{v}| \nabla^2 \mathbf{v} \quad (\nu = 3BL^2 \rightarrow 0^+), \quad (19a)$$

$$\nabla \times \mathbf{v} = \mathbf{0}, \quad (19b)$$

This is a closed set of equations for the velocity field: once solved, this field is used to compute the evolved density field from the continuity equation (13a), much in the same way as with the ZA. The coefficient ν was called the *gravitational multistream* viscosity in Ref. [12]. Because it multiplies the highest-order derivative in Eq. (19a), one cannot simply drop this term in the limit $\nu \rightarrow 0^+$, but one must rather apply the techniques of the boundary-layer theory to study this limit. The mathematical handling is collected in the next subsection; here I simply quote the conclusion that, as expected, the correction regularizes the pancake-like singularities predicted by the dust model, which become robust structures, where more and more mass gets “adhered”. Another conclusion is that this “adhesive” behavior is rather insensitive to the detailed functional dependence on $\nabla \cdot \mathbf{v}$ of the factor in front of $\nabla^2 \mathbf{v}$: hence, the same behavior arises in particular if one simply drops the $|\nabla \cdot \mathbf{v}|$ to obtain a linear correction. But then, one recovers the original adhesion model [7,9,13], which was introduced as a phenomenological (and, when compared against N-body simulations [13], very successful) correction to the ZA to go beyond the epoch of formation of singularities.

In the framework of the large-scale expansion, the physical explanation of the adhesive behavior is clear (Fig. 2). The corrections to dust (18) behave as a force in the opposite direction to the density gradient, so that in the neighborhood of a pancake there is a competition between the inflow of matter driven by the gravitational attraction and this “repulsion”, which prevents the formation of a singularity. The dominant contribution as $\varrho \rightarrow +\infty$ is the correction (18b) due to the velocity dispersion and represents the conversion of “coherent” streaming kinetic energy into “disordered” internal kinetic energy within the coarsening cells [12,14,15] (the mechanism has the same origin as the pressure and viscous-like forces in a gas). The subdominant contribution (18a) from the tidal correction also exhibits this “repulsive” behavior, and means that the macroscopic field \mathbf{w}^{mf} overestimates the gravitational attraction between coarsening cells.

A. Application of the boundary-layer theory

In this subsection I apply the theory of boundary layers [16] to the model (19). The physical picture is that one can study the evolution almost everywhere setting $\nu = 0$. However, there arise regions (“shocks”) spatially well separated from each other and of vanishing volume (as $\nu \rightarrow 0^+$) where \mathbf{v} has a discontinuity, $|\nabla \mathbf{v}|$ diverges and the effect of the term multiplied by ν must be taken into account. The fact that these shocks correspond mainly to plane-parallel structures greatly simplifies the technical handling, since it reduces the problem to the solution of an ordinary differential equation.

The purpose is to study Eqs. (19) in the vicinity of a pancake, so that one takes a point at the pancake and introduces a new coordinate system moving rigidly with it and with one of the axis normal to the pancake. In this new noninertial reference frame Eq. (19a) must be appended with a term \mathbf{A} which collects the acceleration due to the inertial forces, and the velocity \mathbf{v} is now understood as relative to this new reference frame. New rescaled coordinates are defined as $\mathbf{x}' = \mathbf{x}/\varepsilon$ and the idea is to take $\varepsilon \rightarrow 0^+$ in such a way that the fields and their derivatives (and consequently also the thickness of the pancake) remain finite in terms of the new coordinates \mathbf{x}' even in the limit $\nu \rightarrow 0^+$. (The prime of the rescaled coordinates will be dropped hereafter to simplify the notation). Let n , \mathbf{x}_{\parallel} denote the coordinates normal and parallel to the plane of the pancake, respectively, and v_n , \mathbf{v}_{\parallel} the corresponding components of the velocity \mathbf{v} . Let ∇_{\parallel} denote the gradient with respect to \mathbf{x}_{\parallel} . In terms of these new coordinates, Eqs. (19) now read:

$$\varepsilon \frac{\partial \mathbf{v}}{\partial b} - \varepsilon \mathbf{A} - \frac{d\varepsilon}{db} \left(n \frac{\partial}{\partial n} + \mathbf{x}_{\parallel} \cdot \nabla_{\parallel} \right) \mathbf{v} + \left(v_n \frac{\partial}{\partial n} + \mathbf{v}_{\parallel} \cdot \nabla_{\parallel} \right) \mathbf{v} = \varepsilon^{1-\gamma} \nu \left| \frac{\partial v_n}{\partial n} + \nabla_{\parallel} \cdot \mathbf{v}_{\parallel} \right|^{\gamma-2} \left(\frac{\partial^2}{\partial n^2} + \nabla_{\parallel}^2 \right) \mathbf{v}, \quad (20a)$$

$$\left(\mathbf{e}_n \frac{\partial}{\partial n} + \nabla_{\parallel} \right) \times (\mathbf{e}_n v_n + \mathbf{v}_{\parallel}) = \mathbf{0}, \quad (20b)$$

where I have generalized the model (19) by allowing the prefactor of $\nabla^2 \mathbf{v}$ to be an arbitrary function of $\nabla \cdot \mathbf{v}$ whose asymptotic behavior as $\nabla \cdot \mathbf{v} \rightarrow -\infty$ is characterized by the power γ (the case $\gamma = 2$ recovers the original adhesion model). It has also been considered the case that ν (and hence, the rescaling factor ε) may depend on time. The motivation for this generalization is twofold: (i) This kind of behavior corresponds to the models discussed in Refs. [12,14,17], where the hydrodynamic hierarchy (5) is closed by simply neglecting tidal corrections altogether and assuming $\Pi_{ij} = \kappa \varrho^\gamma \delta_{ij}$, $\kappa \rightarrow 0^+$. (ii) The computation of the trace of Π from N-body simulations also yields this kind of behavior [18].

The factor ε must be chosen so as to absorb the explicit dependence on ν in Eq. (20a) and thus to render the coefficient of the highest-order derivative of order unity, namely $\varepsilon = \nu^{1/(\gamma-1)}$ and requiring $\gamma > 1$, so that $\varepsilon \rightarrow 0^+$. This latter constraint states that the correction to dust must grow fast enough with $|\nabla \mathbf{v}|$ so as to successfully oppose the gravitational attraction and overcome the formation of a singularity. Eqs. (20) are now simplified by keeping only the dominant terms in the limit $\varepsilon \rightarrow 0^+$: first, one can set $\nabla_{\parallel} \rightarrow \mathbf{0}$, because in terms of the rescaled coordinates the pancake looks like an infinite plane and the spatial variations along it take place over an infinitely large length scale: only the derivative normal to the pancake is relevant. Also, the acceleration terms $\partial \mathbf{v} / \partial b$ and \mathbf{A} are finite in the neighborhood of pancakes. Finally, $d\varepsilon/db$ is of order ε at any finite time. Therefore, Eqs. (20) reduce to the following system of ordinary differential equations:

$$v_n \frac{\partial v_n}{\partial n} = \left| \frac{\partial v_n}{\partial n} \right|^{\gamma-2} \frac{\partial^2 v_n}{\partial n^2}, \quad (21a)$$

$$\frac{\partial \mathbf{v}_{\parallel}}{\partial n} = \mathbf{0}. \quad (21b)$$

The physical interpretation of these equations is straightforward. The tangential velocity, \mathbf{v}_{\parallel} , is smooth at the pancakes and so constant in the rescaled coordinates. The normal velocity, which looks discontinuous in the physical coordinates, is determined in rescaled coordinates by the balance between the convective transport towards the pancake (governed in turn by the gravitational attraction of the pancake) and the outwards “pressure” due to the velocity dispersion.

The boundary conditions to be imposed to these equations are: (i) $v_n(0) = 0$, meaning that the rescaled coordinate system is centered at the pancake and moves with it; (ii) $\partial v_n / \partial n \leq 0$ for any n , so that there is an inflow of matter towards the pancake (and $\varrho \geq 0$, see Eq. (23)); (iii) $\partial v_n / \partial n \rightarrow 0$ as $n \rightarrow \pm\infty$, so that the derivative in physical coordinates is not divergent outside of the pancake. Integrating twice Eq. (21a) with these boundary conditions, one finds an implicit solution $v_n(n)$:

$$\int_0^{v_n(n)/V} ds (1-s^2)^{\frac{1}{1-\gamma}} = -\frac{\varepsilon n}{\Delta}, \quad \Delta = \varepsilon \left(\frac{\gamma-1}{2} V^{3-\gamma} \right)^{\frac{1}{1-\gamma}}, \quad (22)$$

where $V > 0$ is an integration constant. Knowing $v_n(n)$, one can get an approximation to the pancake density profile by applying the parallelism condition (15) together with Poisson equation (13c) in the limit $\varepsilon \rightarrow 0^+$:

$$\varrho = -\frac{b \varrho_b}{\varepsilon} \frac{\partial v_n}{\partial n}, \quad (23)$$

whose solution reads

$$\varrho(n) = \frac{b \varrho_b V}{\Delta} \left[1 - \frac{v_n(n)^2}{V^2} \right]^{\frac{1}{\gamma-1}}. \quad (24)$$

A detailed study of the solution (22) leads to the general picture shown in Fig. 3: the fields remain univalued at all times and no singularity arises. The parameter $\Delta \rightarrow 0^+$ measures the pancake thickness in the physical coordinates. This study also shows that for the cases $\gamma > 2$ the solution approaches so fast its asymptotic values at large distances that in fact $\partial v_n / \partial n = 0$ at two finite values of the normal coordinate, n_+ and $n_- = -n_+$. In such cases, the solution (22) must be replaced beyond these points by: $v_n(n) = (-)V$, $n < n_-$ ($> n_+$).

From the general solution (22), one can compute the behavior of the solution $v_n(n = z/\varepsilon)$ in the limit $\varepsilon \rightarrow 0^+$ for a fixed value z_0 of the physical coordinate $z = \varepsilon n$:

$$v_n = \begin{cases} V & (\text{if } z < z_0), \\ 0 & (\text{if } z = z_0), \\ -V & (\text{if } z > z_0), \end{cases} \quad (25)$$

independently of the value of γ and the temporal dependence of ν . The conclusion is therefore that the adhesive-like behavior is a robust property of the whole family of models (20) parametrized by γ : whenever the dust evolution ($\nu = 0$) predicts a singularity, this must be replaced by the “adhesive prescription” (25), which leads, via the continuity equation (13a), to a steady accretion of mass.

I have derived and applied a hydrodynamic-like formulation for the process of cosmological structure formation: this is a nontrivial statement, because one could believe that the collisionless nature of the basic model (1) renders a “fluid” description impossible after shell-crossing, and that then one should employ a different approach, e.g. the BBGKY hierarchy. I have shown, however, that a “fluid” description is feasible after the breakdown of the dust model: what makes a difference with “down-to-Earth” fluids is how the closure of the hydrodynamic hierarchy (5) is achieved. For this purpose, I have introduced the large-scale expansion, which builds on the assumption that the large-scale evolution is insensitive to the small scales. This assumption lies behind many reasonings in the cosmological literature: for example, behind the idea that on large scales the evolution follows a Friedmann-Lemaître model, regardless of the small-scale inhomogeneities, and also behind the confidence on cosmological N-body simulations, where each N-body particle is so massive that it must correspond in the real world to a full structure in its own, composed of many smaller particles. The plausibility of the assumption can be argued on the basis of the long range of gravity: the evolution can be expected to be dominated by the large scales provided there is enough large-scale power initially. (One can then also expect that the validity of the large-scale expansion should depend on the initial and boundary conditions). The good agreement with N-body simulations of the truncated ZA and the adhesion model [11,13], in which small-scale structure is completely disregarded, can be viewed as a support of this large-scale dominance.

From the derived hydrodynamic equations, I have shown how the dust model arises as the lowest-order term in the large-scale expansion, while the first-order correction gives rise to a model which can be reduced to the adhesion model by further approximations. This derivation provides a clear physical interpretation of the two models: the “particles” which are assumed to follow the dust model are not such but have an internal structure, and the “interaction” between these “particles” due to its internal structure explains the adhesive behavior. By applying boundary-layer techniques, I found in particular that this behavior is a robust property of the model, in the sense that it arises quite independently of the detailed dependence of the velocity dispersion and the tidal correction on the density and velocity fields.

It is interesting to compare this work with previous, related works dealing also with a hydrodynamic-like formulation of large-scale structure formation. In Refs. [12,19], the case is studied in which the hydrodynamic hierarchy is truncated by a *phenomenological* ansatz which writes the corrections to the adhesion model as a stochastic term (a noise). The adhesion model, in turn, was justified in Refs. [12,14,17] (see Ref. [20] for a relativistic generalization) by disregarding the tidal correction altogether and by letting the velocity dispersion be given as $\Pi_{ij} = \kappa \varrho^\gamma \delta_{ij}$. The robustness of the adhesive property already mentioned explains why this behavior also arises in the models that follow from this truncation. It is particularly instructive to compare with the work in Ref. [14], where a truncation relying on something else than phenomenology was studied. I have been able to close the hydrodynamic hierarchy with less restrictive assumptions and thus shown that the adhesive behavior can still be recovered when assumptions (A2) (isotropic Π) and (A4) and (A5) (further restrictions on the form of Π) in Sec. 4 of Ref. [14] are dropped. Assumption (A1) seems in practice equivalent to the large-scale expansion, while I also employed assumption (A3) (parallelism) in order to derive adhesion-like models. But there is also a major improvement compared to that work: I do not assume the mean-field approximation from the outset (and thus the Vlasov equation, which was the starting point in Ref. [14]). In fact, I could derive an expression for the tidal correction and show that it also behaves “adhesively”. As explained in more detail in App. B, this correction is also the origin for the discrepancy between expression (12) and the relationship $\Pi_{ij} = \kappa \varrho^{5/3} \delta_{ij}$ derived in Ref. [14].

Finally, I have shown that the derivation of adhesion-like models requires in principle further assumptions than the simple large-scale expansion. Thus, one could improve on these models by relaxing those assumptions and rather studying Eqs. (13). An important difference between the adhesion-like models and Eqs. (13) is that the corrections to dust in the latter generate vorticity, even if it is initially absent: this may be a relevant feature when modelling galaxy formation. Another difference is that now there is no need in principle to retain the condition $BL^2 \rightarrow 0^+$: pancakes and other singularities are no longer of vanishing volume, but have an inner structure whose evolution could be studied with Eqs. (13). For such purpose, the role of the length L must be better understood.

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A *smoothing window* $W(z)$ [21] should behave as a *window* that defines a bounded region of space and as a *smoothing* filter that erases the structural details inside this region. These conditions are implemented by requiring $W(z)$ and its Fourier transform $\tilde{W}(q)$ to decay faster than any power as $z \rightarrow \infty$ or $q \rightarrow \infty$, respectively. The smoothing window acts as an integral kernel: if $\phi(\mathbf{x})$ is a given field, then the associated coarse-grained field $\phi_L(\mathbf{x})$ over the scale L and its Fourier transform $\tilde{\phi}_L(\mathbf{k})$ are given by

$$\phi_L(\mathbf{x}) = \frac{1}{L^3} \int d\mathbf{y} W\left(\frac{|\mathbf{x} - \mathbf{y}|}{L}\right) \phi(\mathbf{y}), \quad (\text{A1a})$$

$$\tilde{\phi}_L(\mathbf{k}) = \tilde{W}(L\mathbf{k}) \tilde{\phi}(\mathbf{k}). \quad (\text{A1b})$$

Therefore, the coarse-grained field at point \mathbf{x} is just the (weighted) addition of the original field over a region of size $\approx L$ around that point. Two more conditions are also required: (i) $W(z=0) = 1$, so that the contribution from the neighborhood of the center of the window is unweighted; (ii) $\tilde{W}(q=0) = 1$, so that the coarse-grained field has the same large-scale structure as the original field. This latter condition implies the following normalization:

$$\int d\mathbf{x} W(|\mathbf{x}|) = 1, \quad (\text{A2})$$

so that $\lim_{L \rightarrow 0} L^3 W(|\mathbf{x}|/L) = \delta(\mathbf{x})$, and $\lim_{L \rightarrow 0} \phi_L(\mathbf{x}) = \phi(\mathbf{x})$.

The condition on the decay of $\tilde{W}(q)$ for large q prevents the window $W(z)$ from having sharp borders, which thus becomes a “fuzzy” window. Otherwise, the coarse-grained field $\phi_L(\mathbf{x})$ could change in a discontinuous manner as the center of the window, \mathbf{x} , sweeps the system. In the same way, the fast decay of $W(z)$ for large z (which implies that $\tilde{W}(q)$ lacks sharp borders in Fourier space) implies the property of spatial locality for the coarse-grained field. This also guarantees the existence of the Taylor-expansion of $\tilde{W}(q)$ at $\mathbf{q} = \mathbf{0}$ in the form:

$$\tilde{W}(q) = 1 - \frac{1}{2} B q^2 + o(q^4), \quad (\text{A3})$$

where the constant

$$B = \frac{1}{3} \int d\mathbf{z} z^2 W(z) = \frac{4\pi}{3} \int_0^{+\infty} dz z^4 W(z) \quad (\text{A4})$$

is related to the quadrupole moment of the smoothing window.

The required constraints on the smoothing window exclude a step function (either in real or in Fourier space). A useful function which satisfies the constraints is a Gaussian: $W(z) = \exp(-\pi z^2) \Rightarrow \tilde{W}(q) = \exp(-q^2/4\pi)$.

APPENDIX B: THE EVOLUTION EQUATION FOR Π

In this appendix I study the evolution equation for the velocity dispersion and show that (12) is a solution of the equation. The purpose is to compare with the result in Ref. [14] for Π , also obtained from the evolution equation for this tensor.

Starting from the definition (6) and Eqs. (1a) and (1b), one can obtain the following equation for the temporal evolution of the velocity dispersion:

$$\frac{\partial \Pi_{ij}}{\partial t} + 5H \Pi_{ij} = -\frac{1}{a} \partial_k (u_k \Pi_{ij}) - \frac{1}{a} \Pi_{ik} \partial_k u_j - \frac{1}{a} \Pi_{jk} \partial_k u_i - \frac{1}{a} \partial_k \mathcal{L}_{ijk} + \mathcal{P}_{ij}, \quad (\text{B1})$$

where a summation over repeated indices is implied, and a second-rank and a third-rank tensor fields have been defined:

$$\begin{aligned} \mathcal{P}(\mathbf{x}, t; L) = \int \frac{d\mathbf{y}}{L^3} W\left(\frac{|\mathbf{x} - \mathbf{y}|}{L}\right) \varrho_{mic}(\mathbf{y}, t) \Big\{ [\mathbf{u}_{mic}(\mathbf{y}, t) - \mathbf{u}(\mathbf{x}, t; L)] [\mathbf{w}_{mic}(\mathbf{y}, t) - \mathbf{w}(\mathbf{x}, t; L)] + \\ + [\mathbf{w}_{mic}(\mathbf{y}, t) - \mathbf{w}(\mathbf{x}, t; L)] [\mathbf{u}_{mic}(\mathbf{y}, t) - \mathbf{u}(\mathbf{x}, t; L)] \Big\}, \end{aligned} \quad (\text{B2a})$$

$$\mathcal{L}(\mathbf{x}, t; L) = \int \frac{d\mathbf{y}}{L^3} W\left(\frac{|\mathbf{x} - \mathbf{y}|}{L}\right) \varrho_{mic}(\mathbf{y}, t) [\mathbf{u}_{mic}(\mathbf{y}, t) - \mathbf{u}(\mathbf{x}, t; L)] [\mathbf{u}_{mic}(\mathbf{y}, t) - \mathbf{u}(\mathbf{x}, t; L)]. \quad (\text{B2b})$$

The term \mathcal{L} accounts for the change of velocity dispersion due to the exchange of particles between the coarsening cells, while the term \mathcal{P} represents the change due to the gravitational interaction. To get a physical picture, just consider the equation for the trace of Π (the internal kinetic energy): then \mathcal{L} reduces to the equivalent of the kinetic contribution to the heat flux in the usual hydrodynamics, while \mathcal{P} becomes the power performed by the gravitational interaction.

Compared to Eq. (4c) of [14], Eq. (B1) contains the extra term \mathcal{P} , and the reason is that this term drops if tidal corrections are neglected. Indeed, if one performs a large-scale expansion of \mathcal{P} and \mathcal{L} in the same way as explained in Sec. III, one obtains

$$\mathcal{P} = BL^2 \varrho [(\partial_i \mathbf{u})(\partial_i \mathbf{w}^{mf}) + (\partial_i \mathbf{w}^{mf})(\partial_i \mathbf{u})] + o(L \nabla)^4, \quad (\text{B3a})$$

$$\mathcal{L} = o(L \nabla)^4. \quad (\text{B3b})$$

When these expressions are inserted into Eq. (B1), one can check that the velocity dispersion tensor Π given by (12) is indeed a solution.

In Ref. [14], Eq. (B1) (with $\mathcal{P} = 0$, as explained) was solved for the trace of Π by imposing the constraint of shear-free flow: it is then found that $Tr \Pi = \kappa \varrho^{5/3}$, where κ is determined by the initial velocity dispersion. This must be viewed as a solution valid only for early times; when the tidal corrections grow, the term \mathcal{P} becomes relevant: Π eventually forgets its initial condition, due to the term $5H \Pi$ in Eq. (B1), and becomes “slaved” to the density and velocity fields, as given by Eq. (12). This is a mechanism similar to the “slaving” represented by the parallelism condition (15) in the linearized dust evolution.

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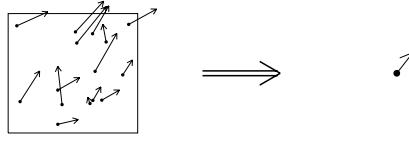


FIG. 1. The dust model amounts to neglecting altogether the internal structure (velocity dispersion and spatial extension) of the coarsening cell: it is approximated by a “big particle” located at the center of the cell, whose mass is that contained within it and whose velocity is the center-of-mass velocity.

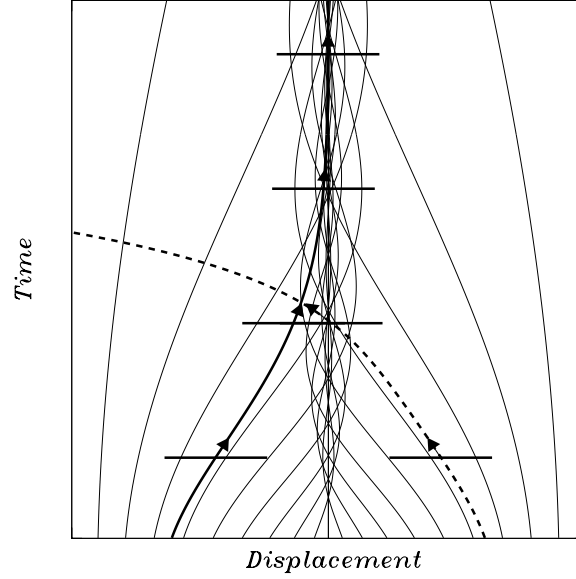


FIG. 2. Sketch of the particle trajectories forming a pancake, based on N-body simulations [22]. The horizontal segment represents a coarsening cell. The dashed trajectory corresponds to the dust model and its extrapolation (by the ZA) beyond the singularity; it is unable to reproduce the stabilization of the pancake. The solid trajectory represents the adhesion model and is more realistic. There is no difference initially, but at the pancake the velocity dispersion becomes very large and produces an effective “adhesive” forcing that corrects the dust evolution.

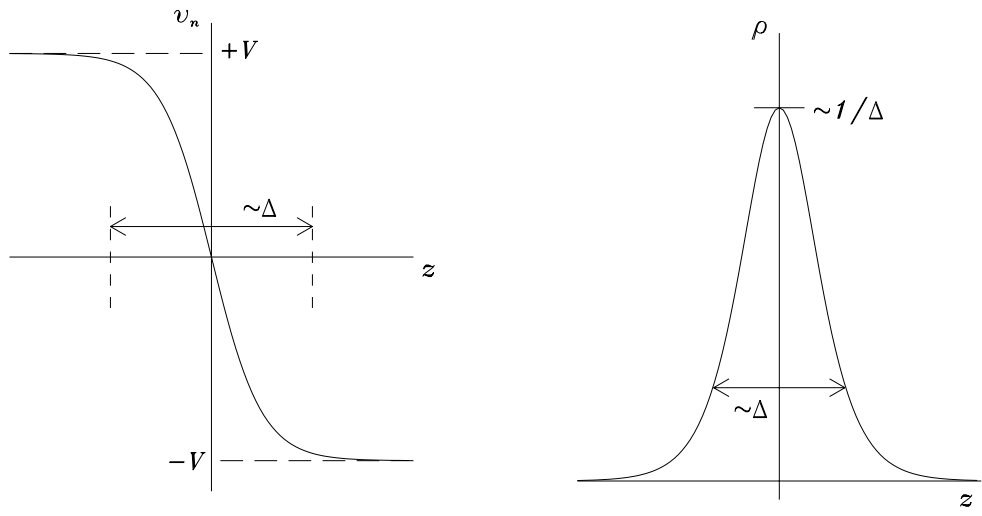


FIG. 3. Qualitative aspect of the coarse-grained density and velocity fields near a pancake according to Eqs. (22) and (24). Δ represents a measure of the pancake thickness.

